M. A./M. Sc. Examination-2021

Semester-IV

Mathematics

Course: MMC-41

(Differential Geometry and Manifold Theory)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. What do you mean by reparametrization of any smooth curve in $\mathbb{R}^3$. Introduce a Frenet frame. State and prove relation among elements of the Frenet frame in the process of reparametrization of a curve. Write down expressions for the above coefficients in terms of derivatives of the position vector at any point on the curve. Find the curvature and torsion of a curve described by $\vec{r}(t) = (\rho \sin\beta \cosh t, \rho \sin\beta \sinh t, \rho\cos\beta), \alpha, \beta \in (0,\pi), t \in \mathbb{R}, \rho = \text{const}$. [1+1+3+3+2]

2. Define a manifold, $\mathfrak{M} \subset \mathbb{R}^3$. State how do you describe a curve Γ on $\mathfrak{M}$. Hence, derive the expressions for the length of a curve Γ and infinitesimal area on a smooth surface ($\mathfrak{M}$) in terms of coefficients of the first fundamental form on $\mathfrak{M}$. Derive expressions for the coefficients of the first fundamental form at any point on the manifold $\mathfrak{M} = \{(x^1, x^2, x^3) \in \mathbb{R}^3 : (x^1)^2 + (x^2)^2 = 1, x^3 \in \mathbb{R}\}$. [1+1+4+4]

3. Introduce normal and geodesic curvature on a manifold, $\mathfrak{M}$ (a smooth surface in $\mathbb{R}^3$). Prove that the normal curvature of the manifold $\mathfrak{M} = \{(x^1, x^2, x^3) \in \mathbb{R}^3 : (x^1)^2 + (x^2)^2 + (x^3)^2 = 1\}$ can be expressed in terms of its first and second fundamental forms. Hence, find the normal curvature of the surface $\vec{r}(s,t) = ((R + \rho\cos s)\cos t, (R + \rho\cos s)\sin t, \rho \sin s), R, \rho = \text{const}$. [2+3+5]

4. Define form, and state its properties. Introduce the concept of contravariant, covariant, and mixed tensor fields with the aid of vector fields and forms. Define wedge product. Find dimension of spaces of two- and three-forms in $\mathbb{R}^3$. Obtain the wedge product $dx^1 \wedge dx^2 \wedge dx^3$ for $x^1 = (R + \rho\cos s)\cos t, x^2 = (R + \rho\cos s)\sin t, x^3 = \rho \sin s, \quad \rho = \text{constant}$. [1+4+5]

5. Define connection form. Explain why it is called form. State and prove the first and second structure equations of Cartan. Find 1-forms and corresponding connection forms on the surface $\vec{r}(s,t) = (\rho \sin s \cos t, \rho \sin s \sin t, \rho \cos s)$. [1+2+4+3]

6. Define covariant derivative of a vector field $\vec{Y}$ with respect to a vector field $\vec{X}$. How do you introduce connection corresponding to moving frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. Prove that connection forms corresponding to an orthonormal frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is skew symmetric. Prove that for the coframe $\theta^1, \theta^2, \theta^3$ corresponding to an orthonormal frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ with connection ω^i_j, $d\theta^i + \omega^i_j \wedge \theta^j = 0$. Find the connection corresponding to the frame $\{\mathbf{e}_\rho, \mathbf{e}_\phi, \mathbf{e}_z\}$ of the cylindrical polar coordinate system (ρ, ϕ, z) of $\mathbb{R}^3$. [1+1+3+3+2]

M. A. / M. Sc. Examination-2021

Semester-IV

Mathematics

Paper: MMC 42

(Operations Research)

Time: 3 Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) What is the criterion for optimal solution in solving a linear programming problem (LPP) by revised simplex method? How can you detect that an LPP has a non-degenerate optimal solution? [1+2]

   (b) Solve the following LPP by revised simplex method clearly pointing out the different steps in the process:

$$\begin{aligned} \text{Maximize } f(x_1, x_2) &= 2x_1 + 3x_2 \\ \text{subject to } x_1 + x_2 &\leq 8, \\ x_1 + 2x_2 &= 5, \\ 2x_1 + x_2 &\leq 8, \\ x_1,\ x_2 &\geq 0. \end{aligned}$$

[7]

2. (a) Justify or correct the statement: Sensitivity analysis has economical importance in the field of operations research. [2]

   (b) Consider the following LPP:

$$\begin{aligned} \text{Minimize} f(x_1, x_2) &= 3x_1 + 5x_2 \\ \text{subject to } x_1 + 2x_2 &\geq 8, \\ 3x_1 + 2x_2 &\geq 12, \\ 5x_1 + 6x_2 &\leq 60, \\ x_1,\ x_2 &\geq 0. \end{aligned}$$

   (a) Solve the LPP.
   (b) Discuss the effect of change of b to $[12, 16, 60]$. [8]

3. Distinguish between integer linear programming problem (ILPP) and LPP. Give examples in support of your answer.
   Hence, find the optimum integer solution of the LPP

$$\begin{aligned} \text{Maximize } f(x_1, x_2) &= 8x_1 + 6x_2 \\ \text{subject to } 8x_1 + 4x_2 &\leq 85, \\ 3x_1 + 6x_2 &\leq 95, \\ x_1,\ x_2 &\geq 0 \text{ and are integers.} \end{aligned}$$

[(2+1)+7]

4. (a) What is quadratic programming problem? Give some application areas of quadratic programming.
   Use the Kuhn-Tucker necessary and sufficient conditions to solve the following nonlinear programming problem (NLPP):

$$\begin{aligned} \text{Maximize } f(x_1, x_2) &= 50x_1 + 15x_2 - 0.5x_1^2 - 10x_2^2 - 6x_1x_2 \\ \text{subject to } 4x_1 + x_2 &\leq 50, \\ x_1 + x_2 &\leq 40, \\ x_1,\ x_2 &\geq 0. \end{aligned}$$

[2+4]

(b) Solve the following NLPP graphically:

$$\begin{aligned} \text{Minimize } f(x_1, x_2) &= (x_1 - 1)^2 + (x_2 - 5)^2 \\ \text{subject to } -x_1^2 + x_2 &\leq 4, \\ x_1 - (x_2 - 2)^2 &\leq 3, \\ x_1,\ x_2 &\geq 0. \end{aligned}$$

[4]

5. (a) Write down the basic assumptions of an Economic Order Quantity (EOQ) purchase model with instantaneous replenishment and with shortages. Hence, show that the mathematical expression of the total annual cost is $C_A^* = \sqrt{\frac{2Dc_s c_1 c_2}{c_1 + c_2}}$; where D, c_s, c_1 and c_2 respectively represents the demand / period, ordering cost / order, carrying cost / unit / period and shortage cost / unit / period. [6]

(b) The purchase manager currently follows EOQ policy of ordering for an item in the stores of his company. The annual demand of the item is 1600 units. Its carrying cost is 40% of the unit cost where the unit cost is Rs. 400. The ordering cost is Rs. 500 per order. Recently, the vendor supplying that item gives a discount of 10% in its unit cost if the order size is minimum of 500 units.
(a) Find EOQ and the corresponding total cost per year.
(b) Check whether the discount offer given by the vendor can be considered by the purchase manager. [2+2]

6. (a) For the queuing model $M/M/1 : N/FIFO$, having Poisson input, exponential service, single service channel, finite capacity of the system, first in first out queue discipline; find the probability of having 0 customer in the system and average queue length of the system. [6]

(b) In a shopping mall, the average arrival rate of customer is 10 in every one hour, following Poisson process. The average time taken by the cashier to list and calculate the customer's purchase is 5 minutes, following exponential distribution.
(i) What is the probablity that the queue length exceeds ten?
(ii) What is the expected time spent by a customer in the system? [2+2]

M. Sc. Examination-2021

Semester-IV

Mathematics

Course: MMC-43

Numerical Computer Laboratory

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Write C-programs for the following problems.
Answer ***any ten*** questions.

1. Find the product of the first n positive even integers using three different loops (for loop/ while loop/ do-while loop). [4]

2. Write a program, which prints the following picture:
   ABCDE
   CDEF
   EFG
   GH
   I [4]

3. Write a program to find the value of the series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots - \frac{x^{50}}{50!}$, where x is the input. [4]

4. Write a program that finds the sum of the even digits. [Example: Input: 23617; Output:2+6=8] [4]

5. Write a program, which reads n numbers and finds the maximum odd number. [Example: Input: $n = 5$; 21, 316, 83, 5, 64. Output: 83] [4]

6. Write a program, which reads two numbers and prints all common factors. [Example: Input: 24, 54; Output: 2,3, and 6] [4]

7. Using for loop read n numbers and evaluate the value of the expression $(x_1 - x_2)(x_2 + x_3) + (x_2 - x_3)(x_3 + x_4) + \ldots$ [4]

8. Write a program, which reads three real numbers a, b, and c as sides of a triangle and prints the area of the triangle. Also, check whether one angle is 90^o or not. [4]

9. Write a program to print the minimum element of every row of an $n \times n$ matrix (First read the matrix). [4]

10. Write a program, which reads two matrices (orders: $n \times m$ and $m \times p$) and finds their product. [4]

11. Write a function, which checks a given number is prime or not. [4]

12. What are the differences between $i++$ and $++i$? Explain with an example. [4]

13. Write a program to print complex numbers 2+3i, 4+5i,..., 10+11i in a file named "ABC". [4]

14. Write a program, which will work as a basic calculator (addition/ subtraction/multiplication/division). [4]

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Semester-IV

Mathematics

Elective Course: MME-41 (Applied Stream)

(Electromagetic Theory and Programming in Matlab)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Electromagnetic Theory)

Answer ***any two*** questions.

1. (a) What is electrostatics? State and explain the Coulomb's law in electrostatics for two point charges. [1+3]

   (b) Define electric field and potential at a point for a charged particle. Calculate the electric field near a long uniformly charged cylindrical rod of radius a. [2+4]

2. (a) Starting from the expression for the electrostatic potential

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\mathbf{r}_0)dV_0}{|\mathbf{r}-\mathbf{r}_0|},$$

   obtain the Poisson law: $\nabla^2\phi = -\rho/\varepsilon_0$. [4]

   (b) Define electrostatic energy of a point charge q in an electric field $\mathbf{E}$. Show that for a continuous charge distribution the energy density of $\mathbf{E}$ is given by $u_e = \frac{1}{2}\varepsilon_0|\mathbf{E}|^2$. [1+5]

3. (a) Derive the equation of continuity for a steady state current of electric charges. What is Ohm's law? Hence show that for a steady state current, the continuity equation reduces to the Laplace equation for the electrostatic potential. [2+1+2]

   (b) Explain the phenomenon of electromagnetic induction and hence establish the differential form of Faraday's law. [5]

Unit-II (Programming in Matlab)

Answer ***any two*** questions.

1. (a) Which command is used to clear a command window? Write the output of the code in editor or command window: $A = [10\ 11; 11\ 10]$; $B = 20$; $C = A + B$. Which function is used to generate a vector x of 10 evenly spaced points in between 0 and 1? [1+2+2]

   (b) How do you find the size of a matrix. Give an example. Which function is used to find the magnitude of a complex number? [(2+1)+2]

2. (a) If $A = [1\ -1\ 2; 2\ -1\ 1; 3\ 0\ 2]$ is a matrix, then write the outputs of (i) A(2,3) (ii) A(:;3). [2]

   (b) Write the Matlab command to find the roots of a polynomial equation $3x^4 - x^2 + 2x - 3 = 0$. [2]

   (c) Write a Matlab program to solve the Lotka-Volterra equations: $\dot{x} = px - qxy,\ \dot{y} = -\alpha y + \beta xy$, where $p,\ q,\ \alpha$ and β are constants. [6]

3. Write a program in MATLAB to solve the following coupled nonlinear equations.

$$x = y + \cos(x+y), \quad y = x + \sin(xy).$$

[10]

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Semester-IV

Mathematics

Optional Course: MMO-41 (A07) (New Syllabus)

(Magnetohydrodynamics-II)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. Derive the fluid equations, namely the equation of continuity and the equation of motion together with the pressure law for a single species charged particle. Also, define the characteristic length and time scales for plasma oscillations. [5+5]

2. Discuss about the electron and ion gyrofrequencies, the characteristic speed and the parameters of approximation for the description of MHD waves in plasmas. [2+2+6]

3. Derive a set of reduced MHD equations for a fully ionized plasma. Also, derive an expression for the generalized Ohm's law for magnetoplasmas. [5+5]

4. Show that for the propagation of MHD waves in a magnetoplasma, the total energy is conserved. [10]

5. (a) Starting from the magnetic field evolution equation, discuss the characteristics of diffusion of magnetic field lines. [5]

   (b) Distinguish between Alfvén waves and magnetosonic waves in a plasma. [5]

6. Write short notes on

   (a) Freezing of magnetic field lines. [4]

   (b) Magnetic viscosity and Reynolds number. [3+3]

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Semester-IV

Mathematics

Optional Paper: MM0-41(P11/A13)

(Weak Formulation of Elliptic Partial Differential Equation)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. Define Dirac delta function, $\delta(x)$. Verify whether this function may be regarded as a test function or a distribution. Determine its nature (regular or singular), if it is a distribution. Express $\delta(x^2-4)$ in terms of $\delta(x \pm a), a \in \mathbb{R}$. Find the value of $\int_{\mathbb{R}}(\sin x + \cos x)\delta'(x^2-\pi^2)$. [1+1+1+3+4]

2. Define Heavyside function, $H(x)$. Express $H(f(x))$ in terms of $H(x)$ for any monotone function $f(x)$. Prove that derivative of distribution follows derivation law. Find the first and second distributional derivatives of $|x|$, if exists. [1+2+2+5]

3. Define classical, weak and distributional solution of a differential equation. Find the solution of the equation $\frac{d^2t}{dx^2}(x) = f(x), 0 < x < 1, t(0) = 0 = t(1)$ for any $f \in \mathcal{D}'$. State the nature (among three mentioned above) of the solution. [3+6+1]

4. What do you mean by a domain Ω in $\mathbb{R}^d$ is a Lipschitz domain? Define Sobolev spaces of integer and real order, and the spaces of functions $W_0^{k,p}(\Omega)$ and $W^{-s,p'}(\Omega)$. Describe the weak formulation of a boundary value problem in some Lipschitz domain Ω involving a second-order linear differential operator on some appropriate function space. State the correspondence rule between the differential operator and the norm of the function space (the domain of the operator involved). [1+4+4+1]

5. When a linear operator involving second-order derivatives is called V-elliptic? State and prove Lax-Milgram lemma. Exercise Lax-Milgram lemma on the problem $u''(x) + u(x) = f(x)$ on $\Omega = (0,1)$ with $u(0) = \alpha, u(1) = \beta$. [1+5+4]

6. Give weak formulation of a general boundary value problem $O_\Omega[u](\bar{x}) = f(\bar{x})$ in a Lipschitz domain $\Omega \subset \mathbb{R}^d$ surrounded by the boundary Γ involving mixed boundary conditions i) $u = 0$ on a part $\Gamma_D \subset \Gamma$, ii) $\sum_{i,j=1}^d a_{i\,j}(\bar{x})\partial_j u(\bar{x})\nu_i = g(\bar{x})$ on the other part $\Gamma_N \subset \Gamma$. Here $\Gamma = \Gamma_D \cup \Gamma_N$ and $\Gamma_D \bigcap \Gamma_N = \Phi$, $O_\Omega \equiv -\sum_{i,j=1}^d \partial_j(a_{ij}(\bar{x})\partial_i) + \sum_{i=1}^d b_i(\bar{x})\partial_i + c(\bar{x})$, $\bar{x} \in \Omega, (\nu_1, \nu_2, \cdots, \nu_d)$ being the unit outward normal to Γ_N. [10]

M. A. / M. Sc. Examination-2021

Semester-IV

Mathematics

Paper: MMO 41 (A05)

(Dynamics of Ecological System-II)

Time: 3 Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. Comment on the effect of discrete time delay in the following interacting species model:

$$\begin{aligned} \frac{du(t)}{dt} &= ru\left(1-\frac{u}{K}\right)-\frac{auv}{b+u}, \\ \frac{dv(t)}{dt} &= \frac{eau(t-\tau)v}{b+u(t-\tau)}-dv, \quad 0<e<1 \\ u(t=0)=u_0(t) &\geq 0, \quad v(t=0)=v_0(t)\geq 0, \quad t\in[-\tau,0], \end{aligned}$$

[10]

where $\tau > 0$ represents the reaction time of the predation.

2. Reflect on the following prey-predator-scavenger model:

$$\begin{aligned} \frac{du}{dt} &= u(1-u)-uv-uw-\alpha_1 u-hu, \\ \frac{dv}{dt} &= \frac{1}{\beta_1}(uv-\alpha_2 v) \\ \frac{dw}{dt} &= \frac{1}{\beta_2}(cuv+uw-\alpha_3 w), \\ u(t=0)=u_0(t) &\geq 0, \quad v(t=0)=v_0(t)\geq 0, \quad w(t=0)=w_0(t)\geq 0, \end{aligned}$$

where u, v and w are the respective population densities of prey, predator and scavenger at time t. Write down your remarks around co-existence equilibrium point on the following issues:

(i) Linear stability analysis,

(ii) Hopf-bifurcation,

(iii) Chaotic dynamics. [3+3+4]

3. Obtain the necessary and sufficient conditions for Turing instability of the following two species interaction:

$$\begin{aligned} \frac{\partial u}{\partial t} &= f_1(u,v)+D_1\nabla^2 u, \\ \frac{\partial v}{\partial t} &= f_2(u,v)+D_2\nabla^2 v, \\ u(0,x)=u_0(x) &\geq 0, \quad v(0,x)=v_0(x)\geq 0, \end{aligned}$$

[10]

where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2}$ and all the system parameters are positive.

4. Consider the reaction-diffusion system

$$\begin{aligned} \frac{\partial u}{\partial t} &= u(1-u)-\frac{uv}{u+\alpha v}+\nabla^2 u, \\ \frac{\partial v}{\partial t} &= \frac{\beta uv}{u+\alpha v}-\gamma v+\delta\nabla^2 v, \\ u(0,x)=u_0(x) &\geq 0, \quad v(0,x)=v_0(x)\geq 0, \end{aligned}$$

where u and v represent concentrations of morphogens at (x,t); $\nabla^2 \equiv \frac{\partial^2}{\partial x^2}$. Find the non-zero equilibrium point of the corresponding temporal system and determine conditions on α, β and γ for a diffusion-driven instability to occur. Calculate the values of δ for which the diffusion-driven instability can take place. [(2+4)+4]

5. Analyse the spatial pattern initiation for the following Schnäkenberg system including diffusion:

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \gamma(\alpha - u + u^2 v) + \nabla^2 u, \\
\frac{\partial v}{\partial t} &= \gamma(\beta - u^2 v) + \delta \nabla^2 v, \\
u(0, x, y) &> 0, \quad v(0, x, y) > 0, \\
\frac{\partial u}{\partial n} &= \frac{\partial v}{\partial n} = 0, \quad (x, y) \in \partial\Omega,
\end{aligned}$$

where u and v represent concentrations of morphogens at (x, y, t); $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; n is the outward unit normal vector of the smooth boundary $\partial\Omega$ of the reaction-diffusion domain Ω. [10]

6. Calculate traveling-wave front solutions of the following reaction-diffusion prey-predator system:

$$\begin{aligned}
\frac{\partial u}{\partial t} &= ru\left(1 - \frac{u}{K}\right) - \frac{au^2 v}{b + u^2} + D_1 \frac{\partial^2 u}{\partial x^2}, \\
\frac{\partial v}{\partial t} &= -dv + \frac{cu^2 v}{b + u^2} + D_2 \frac{\partial^2 v}{\partial x^2},
\end{aligned}$$

where u representing prey population size; v representing predator population size; and all other system parameters are positive. [10]

M. A./M. Sc. Examination 2021
Semester-IV
Mathematics
Optional Course: MMO-41(P10/A12)
(Theory of Computation-II)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Answer *any four* questions.

1. (a) Show that the language
$$L = \{a^n b^n \mid n \geq 1\}$$
is not regular. [2]

(b) Show that if L is a regular set over Σ then $\Sigma^* - L$ is also so. [2]

(c) Show that every context-free language is also context-sensitive. [2]

(d) Show that if a pushdown automaton A accepts L by final state then there exists a pushdown automaton B accepting L by final store. [2]

(e) Define a Turing machine. Describe the representation of a Turing machine by instantaneous description. [2]

2. (a) Define regular expression and its valuation. Show that every regular expression corresponds to a non-deterministic finite automaton. [1+2]

(b) Construct a minimum state deterministic finite automaton corresponding to the regular expression [5]
$$(a^* \mid b^*)bab.$$

(c) Construct a regular expression corresponding to the transition system given by the following table: [2]

Table 1: Transition Table

	0	1
$\rightarrow a_0$	a_0	a_1
a_1	a_2	a_1
$a_2 \leftarrow$	a_2	a_2

3. (a) What is meant by derivation in a CFG? Show that if A derives w in G then there is a leftmost derivation of w. [1+2]

(b) Let $G =< V_N, V_T, P, S >$ be given by the productions $\{S \rightarrow AB, A \rightarrow a, B \rightarrow b, B \rightarrow C, D \rightarrow c\}$. Find grammar G' such that every variable in G' derives some terminal string. [3]

(c) Construct a reduced grammar equivalent to the grammar whose productions are given by $\{S \rightarrow aAa, A \rightarrow Sb \mid bCC \mid DaA, C \rightarrow abb \mid DD, D \rightarrow aDA, E \rightarrow ac\}$. [4]

4. (a) Define a pushdown automaton. What is meant by an instantaneous description of a pushdown automaton? Explain the working of a pushdown automaton by using change in instantaneous descriptions. [1+1+1]

(b) Design a pushdown automaton accepting the language [3]
$$L = \{a^m b^n \mid m > n \geq 1\}.$$

(c) Show that if L is a context-free language, then there exists a pushdown automaton M such that $L = N(M)$, the language accepted by empty store. [4]

5. (a) Define Turing machine. A Turing machine is described by the following transition table

Table 2: Transition Table

	0	1	b
$\rightarrow q_1$	$0Rq_1$		$1Lq_2$
q_2	$0Lq_2$	$1Lq_2$	bRq_3
q_3	bRq_4	bRq_5	
q_4	$0Rq_4$	$1Rq_4$	$0Rq_5$
$q_5 \leftarrow$			$0Lq_2$

Write down the computation sequence of the input string $00b$. [2+3]

(b) Design a Turing machine M to recognize the language [5]

$$\{a^n b^n c^n \mid n \geq 1\}.$$

6. (a) Define primitive recursive function over N. Show that the function $f_1(x, y) = x + y$ is primitive recursive. [1+1]

(b) Define recursive function over N. Show that $f_2(x) = x/2$ is a partial recursive function. [1+1]

(c) What is a Turing model for computation? Design a Turing machine for computing the successor function $S(x)$. Use it to show that $S(1) = 2$. [2+3+1]