Four Year Undergraduate Examination, 2025
Semester-IV
Mathematics (Major)
Course: MJMA 07
(Algebra-IV)

Time: Three Hours **Full Marks: 80**

Questions are of values indicated in the margin. Notation and symbols have their usual meanings.

Answer question ***no 1*** and ***any four*** of the rest.

1. Answer ***any ten*** questions. Provide a clear and complete justification for each answer whenever necessary. Each question carries 2 marks. **$(10 \times 2 = 20)$**

   (a) Give an example to show that a subfield K in a field F may not be a vector space over F.

   (b) Is $W = \{(a_1, a_2, 0) : a_1, a_2 \in \mathbb{R}\}$ isomorphic to $\mathbb{R}^2$?

   (c) Determine whether the set $\{x^3 - x,\ 2x^2 + 4,\ -2x^3 + 3x^2 + 2x + 6\}$ is linearly dependent or independent in $\mathcal{P}_3(\mathbb{R})$:

   (d) Give an example of a vector space having 121 elements.

   (e) Is the vector space of real numbers $\mathbb{R}$ over the field of rational numbers $\mathbb{Q}$ finite-dimensional?

   (f) Find the row rank of the matrix $\begin{pmatrix} 10 & 0 & -8 \\ 2 & -4 & 3 \\ -5 & 7 & 6 \end{pmatrix}$.

   (g) Prove or disprove: If a linear transformation $T : V \to W$ is one-to-one, then the only vector $x \in V$ such that $T(x) = \theta_W$ is $x = \theta_V$.

   (h) Check whether the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(a_1, a_2) = (|a_1|, a_2)$ is linear or not.

   (i) Find $T(a, b)$, where $T : \mathbb{R}^2 \to \mathbb{R}^2$ represents the projection onto the y-axis along the x-axis.

   (j) Let V be a vector space with the ordered basis $\beta = \{v_1, v_2, \ldots, v_n\}$. Define $v_0 = \theta_V$, and let $T : V \to V$ be a linear transformation such that $T(v_j) = v_j + v_{j-1}$, for $j = 1, 2, \ldots, n$. Find the matrix representation $[T]_\beta$.

   (k) What is the necessary condition on m and n such that the vector spaces $\mathcal{P}_n(F)$ and $\mathcal{P}_m(F)$ are isomorphic, where F is a field?

   (l) Does there exist a singular change-of-basis matrix from one ordered basis B_1 to another ordered basis B_2 in a vector space?

   (m) Can a finite-dimensional vector space have a dual space of a different dimension?

   (n) Let $\beta = \{(2, 1), (3, 1)\}$ be an ordered basis for $\mathbb{R}^2$. Find two linear functionals f_1 and f_2 that form a minimal generating set for the dual space $(\mathbb{R}^2)^*$.

   (o) Can the row vectors of a nonsingular matrix A form a basis for the row space of A?

2. (a) Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2) \quad \text{and} \quad c(a_1, a_2) = (ca_1, ca_2).$$

   Is V a vector space over $\mathbb{R}$ with these operations? Justify your answer. [3]

   (b) Prove that $W_1 = \{(a_1, a_2, \ldots, a_n) \in \mathbb{F}^n : a_1 + a_2 + \cdots + a_n = 0\}$ is a subspace and $W_2 = \{(a_1, a_2, \ldots, a_n) \in \mathbb{F}^n : a_1 + a_2 + \cdots + a_n = 1\}$ is not a subspace. [3+2]

   (c) Let W be a subspace of a vector space V over a field $\mathbb{F}$. Prove that the set $S = \{v + W : v \in V\}$ with respect to addition and scalar multiplication by scalars of F given by $(v_1 + W) + (v_2 + W) = (v_1 + v_2 + W)$ and $a(v_1 + W) = av_1 + W$ for all $a \in \mathbb{F}$ is a vector space over F. [7]

3. (a) Give an example of three linearly dependent vectors in R^3 such that none of the three is a multiple of another. [3]

(b) Prove that $L(\{x\}) = \{ax : a \in \mathbb{F}\}$ for any vector x in a vector space over a field F. Interpret this result geometrically in R^3. [3+2]

(c) Let $S = \{(1,1,0),(1,0,1),(0,1,1)\}$ be a subset of the vector space $\mathbb{F}^3$. [3]

i. Prove that if $\mathbb{F} = \mathbb{R}$, then S is linearly independent. [4]

ii. Prove that if $\mathbb{F}$ has characteristic 2, then S is linearly dependent.

4. (a) Let V be a vector space with dimension n. Show that every linearly independent subset of V can be extended to a basis for V. [5]

(b) Find a basis for the subspace W of $M_{n\times n}(F)$ of all upper triangular $n \times n$ matrices. What is the dimension of W? [3+1]

(c) Consider the subspaces $W_1 = \left\{ \begin{bmatrix} a & b \\ c & a \end{bmatrix} \in V : a,b,c \in \mathbb{F} \right\}$, and $W_2 = \left\{ \begin{bmatrix} 0 & a \\ -a & b \end{bmatrix} \in V : a,b \in \mathbb{F} \right\}$ of $V = M_{2\times 2}(\mathbb{F})$. Find the dimensions of W_1, W_2, $W_1 + W_2$, and $W_1 \cap W_2$ respectively. [2+2+1+1]

5. (a) Prove that there exists a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1,1) = (1,0,2)$ and $T(2,3) = (1,-1,4)$. What is $T(8,11)$? [3+1]

(b) Find the rank and nullity of the linear mapping $G : \mathbb{R}^5 \to \mathbb{R}^3$ defined by $G(x,y,z,s,t) = (x+2y+2z+s+t, x+2y+3z+2s-t, 3x+6y+8z+5s-t)$. Is the mapping nonsingular? Justify your answer. [3+3+1]

(c) Suppose V is a vector space of dimension n and T be a linear oparetor on V with $\text{rank}T^2 = \text{rank}T$. Then find $\ker T \cap \text{Im}T$. Hence, show that V is a direct sum of $\ker T$ and $\text{Im}T$. [3+1]

6. (a) Let T be a linear operator on a finite-dimensional vector space V, and let β and β' be two ordered bases for V. Suppose that Q is the change of coordinate matrix that changes β-coordinates into β'-coordinates. Then show that $[T]_{\beta'} = Q^{-1}[T]_\beta Q$. [5]

(b) Let V and W be vector spaces, and let $T : V \to W$ be a linear and invertible transformation. Show that $T^{-1} : W \to V$ is linear. For the linear transformation $T : \mathcal{P}_1(\mathbb{R}) \to \mathbb{R}^2$ defined by $T(a+bx) = (a, a+b)$, find the formula for T^{-1}. [2+1]

(c) Let $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ be the linear transformation defined by $T(f(x)) = f'(x)$, and let β and γ be the standard ordered bases for $\mathcal{P}_3(\mathbb{R})$ and $\mathcal{P}_2(\mathbb{R})$, respectively. For the polynomial $p(x) = 2 - 4x + x^2 + 3x^3 \in \mathcal{P}_3(\mathbb{R})$, verify that $[T(p(x))]_\gamma = [T]^\gamma_\beta [p(x)]_\beta$. [7]

7. (a) For subspaces W_1 and W_2 in a vector space V over a field F, show that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$. [4]

(b) Let $V = \mathbb{R}^3$, and consider the basis $\{f_1(x,y,z) = x-2y, \quad f_2(x,y,z) = x+y+z, \quad f_3(x,y,z) = y - 3z\}$ of V^*. Then find a basis for V for which $\{f_1, f_2, f_3\}$ is the dual basis. [4]

(c) Consider the vector spaces $V = \mathcal{P}_1(\mathbb{R})$ and $W = \mathbb{R}^2$ with respective standard ordered bases β and γ. Define $T : V \to W$ by $T(p(x)) = \big(p(0) - 2p(1), p(0) + p'(0)\big)$, where $p'(x)$ is the derivative of $p(x)$. Compute $[T^t]^{\gamma^*}_{\beta^*}$. [7]

Four Year Undergraduate Programme - 2025

Semester-IV(NEP)
Mathematics
Paper: MJMA-08
(Analysis IV)

Time: 3 Hours **Full Marks: 80**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meaning.

Answer ***any eight*** questions.

1. (a) Define pointwise and uniform convergence of a sequence of functions $\{f_n\}$. [2]
   (b) If $f_n(x) = n^2x^2e^{-nx}$, determine whether the sequence $\{f_n\}$ is uniformly convergent on $[0, \infty)$. [4]
   (c) Show that the sequence $\{\tan^{-1} nx\}$ is pointwise convergent on $\mathbb{R}$. [4]

2. (a) Let $\{f_n\}$ be a pointwise convergent sequence of bounded functions on $D \subseteq \mathbb{R}$. Show that its limit function may not be bounded on D. [4]
   (b) If a sequence $\{f_n\}$ of Riemann integrable functions converges uniformly to a function f on a closed and bounded interval $[a, b]$, show that f is Riemann integrable on $[a, b]$ and the sequence $\left\{\int_a^b f_n\right\}$ converges to $\int_a^b f$. [6]

3. (a) State and prove Cauchy's principle for uniform convergence of series of functions. [4]
   (b) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\cos nx}{k^2 - n^2}$ (where $k \in \mathbb{R} \setminus \mathbb{Z}$) is uniformly convergent on any closed and bounded interval. [3]
   (c) Check if the series $1 + x + x^2 + x^3 + \cdots$ is uniformly convergent on $[0, 1)$. [3]

4. (a) Let a series $\sum f_n$ of continuous functions be uniformly convergent on $D \subseteq \mathbb{R}$. Prove that its sum function is continuous on D. [4]
   (b) Show that the series $\sum_{n=1}^{\infty}(-1)^n(x^n - x^{n+1})$ converges uniformly on $[0, 1]$ but $\sum_{n=1}^{\infty}(x^n - x^{n+1})$ does not converge uniformly on $[0, 1]$. [3+3]

5. (a) If a power series $\sum_{n=0}^{\infty} a_n x^n$ diverges for $x = c$, show that it diverges for all $x \in \mathbb{R}$ with $|x| > |c|$. [4]
   (b) State Cauchy-Hadamard Theorem for power series. If $\sum a_n$ is an absolutely convergent series of real numbers, then show that $\sum_{n=0}^{\infty} a_n x^n$ converges absolutely on $[-1, 1]$. [2+4]

6. (a) Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n^5+1)}{(n^7+1)3^n} x^n$. [4]
   (b) What do you mean by a real analytic function? Explain with example. [3]
   (c) Prove or disprove: All functions which are infinitely many times differentiable on an interval are real analytic functions. [3]

7. (a) Prove that if $\lim_{(x,y)\to(a,b)} f(x, y)$ exists finitely then $f(x, y)$ is necessarily bounded in some deleted neighbourhood of the point (a, b). [3]
   (b) Using $\epsilon - \delta$ definition, show that $\lim_{(x,y)\to(0,0)} \frac{x^2y^4}{x^2 + y^2} = 0$. [3]
   (c) Prove that if the double limit $\lim_{(x,y)\to(a,b)} f(x, y) = l$ and one repeated limit $\lim_{x\to a}\lim_{y\to b} f(x, y) = l'$ exists then $l = l'$. [4]

8. (a) Show that the function $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$ is continuous at $(0, 0)$. [3]

(b) Prove that if $f(x,y)$ is continuous at a point (a,b) and $f(a,b) > 0$, then $f(x,y) > 0$ in some neighbourhood of (a,b). [2]

(c) When a function $f(x,y)$ is said to be differentiable at a point (a,b)? If $f(x,y)$ is differentiable at a point (a,b), then show that the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ both exist finitely at (a,b). [1+4]

9. (a) State and prove a sufficient condition for differentiability of a function of two variables at a point (a,b). [4]

(b) State and prove Young's theorem for equality of two mixed second order partial derivatives of a function $f(x,y)$ at a point (a,b). [6]

10. (a) Let $f(x_1,x_2)$ be differentiable in a region R and $x_1\frac{\partial f}{\partial x_1} + x_2\frac{\partial f}{\partial x_2} = nf(x_1,x_2)$ for (x_1,x_2) in R. Then show that $f(x_1,x_2)$ is a homogeneous function of degree n in R. [4]

(b) State and prove Mean Value Theorem for a function of two variables. [4]

(c) Expand $f(x,y) = xy^2 + 3x - 2$ in powers of $(x+1)$ and $(y-2)$. [2]

11. (a) State and prove Implicit function theorem for a function of two variables. [5]

(b) Prove that a necessary condition for a function $f(x,y)$ to have an extreme value at (a,b) is that $\frac{\partial f(a,b)}{\partial x} = 0, \frac{\partial f(a,b)}{\partial y} = 0$, provided these partial derivatives exist. [3]

(c) Show that $f(x,y) = x^4 + x^2y + y^2$ has a minimum at $(0,0)$. [2]

12. (a) Use Lagrange method to find the shortest distance from the point $(2,0)$ to the parabola $y^2 = 8x$. [4]

(b) Evaluate $\int\int_R xy^2(x^2+y^2)dxdy$ over $R : \{(x,y) : 0 \le x \le a; 0 \le y \le b\}$. [3]

(c) Evaluate $\int\int_E x^2y^2dxdy$ over the region E bounded by $x \ge 0, y \ge 0$ and $x^2 + y^2 \le 1$. [3]

Use separate answer scripts for each unit

Four Year Undergraduate Examination-2025

Semester-IV (NEP)
Mathematics (Major)
Paper: MJMA 09
(Differential Equations III and Dynamics of a Rigid Body)

Time: Three Hours **Full Marks: 80**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I [Differential Equations III (Marks: 40)]

Answer question number 1 and ***any two*** from the rest.

1. Answer ***any five*** questions.

(a) When is a partial differential equation (PDE) considered well-posed in Hadamard's sense?

(b) Form a PDE by eliminating the arbitrary function ϕ from

$$u = x^2 + y^2 + \phi\left(\frac{1}{x} + \log_e y\right).$$

(c) Distinguish between linear, semilinear and quasilinear types of first order PDE in two independent variables.

(d) Show that the PDE

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = u^2$$

has solutions of the form $u = c_1 e^x$ and $u = c_2 e^y$, where c_1 and c_2 are two independent arbitrary constants, but their sum $c_1 e^x + c_2 e^y$ is not a solution of the equation. Justify your answer.

(e) Find the integral surface of the Cauchy problem

$$\frac{\partial u}{\partial x} - 3\frac{\partial u}{\partial y} = 0,\ u(x,0) = -x^3.$$

(f) Define the compatibility of two first-order PDEs in two independent variables.

(g) Find a singular solution of the following non-linear PDE, if any:

$$\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\left(u - x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y}\right) = 1.$$

(h) Find the solution of

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} = 0,$$

such that u is the product of a function of x and a function of y. [5 × 2=10]

2. (a) Find the general solution of the following first order quasi-linear PDE:

$$P(x,y,u)\frac{\partial u}{\partial x} + Q(x,y,u)\frac{\partial u}{\partial y} = R(x,y,u).$$

Hence, comment on the uniqueness of the obtained integral surface with given initial curve $\Gamma : \{x_0(s), y_0(s), u_0(s)\}$.

(b) Show that the orthogonal surface passing through the hyperbola $x^2 - y^2 = 9, u = 0$ to the one-parameter family of surface represented by $u = Cxy(x^2 + y^2)$ is

$$(x^2 - y^2)^2(x^2 + y^2 + 4u^2) = 81(x^2 + y^2),$$

where C is a constant. [(6+2)+7]

3. (a) Let $u(x,y) = 2f(y)\cos(x-2y), (x,y) \in \mathbb{R}^2$, be a solution of the initial value problem (IVP)

$$2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u, u(x,0) = \cos x.$$

Find the value of $f(1)$.

(b) Prove that the PDEs

$$\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] y = u\frac{\partial u}{\partial y} \text{ and } \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 2025,$$

are compatible and hence find the one-parameter family of solutions. Furthermore, identify the domain in which the one-parameter family of solutions is applicable.

(c) Does any singular integral exist for a first-order PDE of the form $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0$? Justify your answer.

[5+(2+4+2)+2]

4. (a) Check whether $u = c_1 y + c_2(x^2 - c_1)$ is a complete integral or not of the PDE

$$2xu - x^2\left(\frac{\partial u}{\partial x}\right) - 2xy\left(\frac{\partial u}{\partial y}\right) + \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) = 0,$$

where c_1 and c_2 are two independent arbitrary constants.

(b) Find the complete integral of the following PDE using Charpit's method:

$$x^2\left(\frac{\partial u}{\partial x}\right)^2 + y^2\left(\frac{\partial u}{\partial y}\right)^2 = u^2.$$

(c) Solve:

$$\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + 9\frac{\partial^2 u}{\partial y^2} - 4\frac{\partial u}{\partial x} + 12\frac{\partial u}{\partial y} + 4u = e^{2x}\tan(3x+y).$$

[2+7+6]

Unit-II [Dynamics of a Rigid Body (Marks: 40)]

Answer ***any four*** questions.

1. State the principle of parallel axes. Using this principle, obtain the M.I. of a circular disc about a tangent parallel to the diameter. [3+7]

2. (a) Show that the momental ellipsoid at a point on the edge of the circular base of a thin hemispherical shell is given by the equation $2x^2 + 5(y^2 + z^2) - 3xz = constant$. [5]

   (b) To find whether a given straight line is, at any point of its length, a principle axis of a material system. [5]

3. (a) Using D'Alembert's principle, obtain the equation of motion of the centre of inertia. [3]

   (b) A rough uniform board of mass M and length $2l$, rests on a smooth horizontal plane, and a man of mass m walks on it from one end to the other. Show that the distance through which the board moves is $\frac{2ml}{m+M}$. [7]

4. (a) Obtain the kinetic energy of a rigid body rotating about an axis. [3]

   (b) A vertical thread upwinds itself from a reel, the upper end of the thread being fixed. Prove that the downward acceleration of the reel is $\frac{a^2}{a^2+k^2} \times g$ and that the tension of the thread is $\frac{k^2}{a^2+k^2} \times Mg$, where M is the mass of the reel, a is its radius and k is the radius of gyration about that axis. [7]

5. (a) Prove that the centres of oscillation and suspension are interchangeable. [3]

   (b) A uniform elliptic board swings about a horizontal axis at right angle to the board and passing through one focus. If the centre of oscillation be at the other focus, the prove that the eccentricity of the ellipse is $\sqrt{\frac{2}{5}}$. [7]

6. (a) Prove that the kinetic energy of a rigid body moving in two dimensions is $\frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\dot{\theta}^2$. [5]

   (b) A circular cylinder of radius a and radius of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane through their axes moves like a circular pendulum of length

$$(a-b)\left(1+\frac{k^2}{a^2}\right).$$

[5]

Four-Year Undergraduate Examination-2025

Semester-IV (NEP)

Mathematics (Major)

Paper: MJMA-10

(Mathematical Probability and Statistics)

Time: Three Hours **Full Marks: 80**

Questions are of values as indicated in the margin.
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Unit-I (Full Marks: 40)
(Mathematical Probability)

Answer ***any four*** questions.

1. (a) State the axioms of mathematical probability. [3]

   (b) If $\{A_n\}$ is a monotonic non-decreasing sequence of events, then prove that [7]

$$P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n).$$

2. (a) Prove the general addition rule of probability for any two events A_1 and A_2. [3]

   (b) A card is drawn at random from each of two well-shuffled packs of cards. What is the probability that at least one of them is queen of spades? [7]

3. (a) State Bayes' theorem. [3]

   (b) Two urns contain respectively 2 white and 1 black balls, and 1 white and 5 black balls. One ball is transferred from the first to the second urn, and then a ball is drawn from the second urn. What is the probability that the ball drawn is white? [7]

4. (a) Define the distribution function $F(x)$ of a random variable X. Prove that for any $a < b$, $F(a) \leq F(b)$. [2+3]

   (b) Let the spectrum of the random variable X be $\{1, 2, ..., n\}$ and $P(X = i) \propto 1/i(i+1)$. Determine the distribution function of X and compute $P(3 < X \leq n)$. [3+2]

5. (a) If a random variable X is uniformly distributed over the interval $(0, \pi/2)$, compute the expectation of the function $g(x) = \sin X$. [3]

   (b) A continuous random variable X has probability density $f(x) = \beta e^{-\beta x}$ with $0 < x < \infty$ and $\beta > 0$. Calculate the moment generating function of X, and hence obtain the k-th order moment of X about the origin (α_k). [2+5]

6. (a) If $\rho(X, Y)$ denotes the correlation coefficient of two random variables X and Y, prove that $-1 \leq \rho(X, Y) \leq 1$. [3]

   (b) State Tchebycheff's inequality. Using it show that in 2000 throws with a coin, the probability that the number of heads lies between 900 and 1100 is at least 19/20. [2+5]

Unit-II (Full Marks: 40)
(Mathematical Statistics)

Answer ***any four*** questions.

7. Define a population and a random sample. What is meant by the distribution of a random sample? Show that if the size of the sample is large, the distribution of the sample is the statistical image of the distribution of the population. [2+2+6]

8. Define an estimate of a population parameter. When is an estimate said to be consistent and unbiased? For a normal (m, σ) population, show that the sample variance S^2 is a consistent but biased estimate of σ^2. Obtain an estimate of σ^2 that is both consistent and unbiased. [2+2+(5+1)]

9. (a) Define a *statistic*. Find the sampling distribution of the mean for the binomial (N, p) population. [1+4]

   (b) Estimate the parameter α of a continuous population having the probability density function $f(x) = (1+\alpha)\, x^{\alpha}$ $(0 < x < 1)$ by the method of maximum likelihood. [5]

10. (a) Define a confidence interval of a population parameter. What is confidence coefficient? [2+1]

    (b) The population of scores of 10-year old children in a psychological performance test is known to have a standard deviation 5.2. If a random sample of size 20 shows a mean of 16.9, find 95% confidence limits for the mean score of the population, assuming that the population is normal. Given $P(U > 1.96) = 0.025$, where U is a standard normal variate. [7]

11. (a) Define (i) Simple hypothesis, (ii) Composite hypothesis, (iii) Null hypothesis, and (iv) Alternative hypothesis, and give examples. [2+2+2+2]

    (b) What are meant by the rejection and acceptance regions of testing a statistical hypothesis? [2]

12. (a) Define two types of errors in testing a statistical hypothesis. [2+2]

    (b) Let p be the probability that a coin will fall head in a single toss in order to test the null hypothesis $H_0 : p = 1/2$ against the alternative $H_1 : p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probabilities of two types of errors. [3+3]