Use separate answer script for each unit

# B. Sc. (Honours) Examination-2025

**Semester-VI (CBCS)**
**Mathematics**
**Course: CCMA-13**
**(Analysis-VI)**

**Time: 3 Hours** **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

## Unit-I (Full Marks: 30)
(Complex Analysis)

Answer ***any three*** questions.

1. (a) Prove that a necessary and sufficient condition that a function $f(z) = u(x,y) + iv(x,y)$ tends to a complex number $l = \alpha + i\beta$ as $z = x + iy$ tends to $z_0 = a + ib$ is that $\lim_{(x,y)\to(a,b)} u(x,y) = \alpha$ and $\lim_{(x,y)\to(a,b)} v(x,y) = \beta$. [4]

   (b) Let $f$ be an entire function such that $f(z) = u(y)+iv(x)$. Show that $f$ can be expressed as $f(z) = az+b$, for some $a, b \in \mathbb{C}$ with $\mathrm{Re}(a) = 0$. [3]

   (c) Verify Cauchy-Riemann equations for the function $f(z) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} + i\frac{x^3+y^3}{x^2+y^2} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$ at $z = 0$.
   Is $f$ differentiable at $z = 0$? Justify your answer. [3]

2. (a) Let $f(z) = u + iv$ be an entire function such that $v$ is bounded. Prove or disprove: $f$ is constant. [3]

   (b) Let $f(z) = u + iv$ be an analytic function on a domain $G$. Find $f(z)$ explicitly if $2u + 3v = 13(x^2 - y^2) + 2x + 3y$. [3]

   (c) Describe stereographic projection of the complex plane $\mathbb{C}$ and obtain the formulae for the point $(\xi, \eta, \zeta)$ on the Riemann sphere corresponding to a given point $z = x + iy$ in $\mathbb{C}$. [4]

3. (a) Find all the Möbius transformations of the unit disc $|z| \leq 1$ onto the upper half-plane $\mathrm{Im}(w) \geq 0$. [4]

   (b) Show that the series $\sum_{n=1}^{\infty} \frac{i^n}{n}$ is conditionally convergent. [3]

   (c) Prove that a Möbius transformation maps a circle onto a circle and inverse points onto inverse points. [3]

4. (a) Let $f$ be analytic in a domain $D$ and $f'(a) \neq 0$ for some $a \in D$. Prove or disprove: $f$ is one-one in some neighbourhood of $a$. [3]

   (b) State orientation principle and use to find the image of the interior of thr circle $|z + 3| = 3$ under the Möbius transformation $w(z) = \frac{z}{2z+6}$. [4]

   (c) Find the integral of the function $f(z) = x + y - 3ix^3$ along the curve $y = x$ from the point $z = 0$ to the point $z = 1 + i$. [3]

5. (a) State and prove Cauchy's Integral Formula for derivative. [4]

   (b) Let $f$ be analytic in a domain $G$. Prove that all the derivatives of $f$ exist and are analytic in $G$. [3]

   (c) Evaluate $\oint_C \frac{e^{iz}}{z(z-\pi)} dz$, where $C$ is the (positively oriented) circle $|z| = 3$. [3]

## Unit-II (Metric Spaces)(Marks: 30)
Answer ***any three*** questions.

6. (a) Define a 'Discrete metric space'. [1]

   (b) Prove that for any three points $x, y, z$ in a metric space $(X, \rho)$, $|\rho(x,y) - \rho(z,x)| \leq \rho(y,z)$. [2]

   (c) Let $(X, \rho)$ be a metric space and $d(x,y) = \frac{3\rho(x,y)}{1+3\rho(x,y)}$. Examine if $(X, d)$ is a metric space. [3]

   (d) Prove that $C[a,b]$, the set of all real valued continuous functions on $[a,b]$, is a metric space with respect to a metric defined by you. [4]

7. (a) Show that arbitrary union of open sets is open in a metric space. [3]

   (b) Is the arbitrary intersection of open sets open? Justify your answer. [1+2]

   (c) Define the closure $\bar{A}$ of a set $A$ in a metric space $(X, \rho)$. Prove that if $\alpha \in \bar{A}$ then $S \cap A \neq \phi$ for every neighbourhood $S$ of $\alpha$. [1+3]

8. (a) Define a convergent sequence in a metric space $(X, \rho)$. If $x_n \to \alpha$ and $y_n \to \beta$ as $n \to \infty$, then show that $\rho(x_n, y_n) \to \rho(\alpha, \beta)$ as $n \to \infty$. [1+2]

   (b) Prove that a Cauchy sequence in a metric space, which has a convergent subsequence, is itself convergent. [3]

   (c) Show that $l_p (p \geq 2)$ is a complete metric space. [4]

9. (a) Let $X$ be a complete metric space and $Y$ be a subset of $X$. Then prove that $Y$, considered as a metric space is closed if and only if it is complete. [2+2]

   (b) Define a set of first category and a set of second category in a metric space. [2]

   (c) Define a fixed point of a linear operator $T$ in a metric space $(X, \rho)$. Let $X = (0, \frac{1}{9})$ be a metric space with the usual metric of $R$. Let $T : X \to X$ be given by $Tx = x^2$. Show that T is a contraction mapping having no fixed point. [1+3]

10. (a) State and prove Banach fixed point theorem. [1+4]

    (b) Let $(X, \rho)$ and $(Y, \rho_1)$ be two metric spaces and $f : X \to Y$ be a function. Then $f(x)$ is continuous at $x \in X$ if and only if for every sequence $\{x_n\}, x_n \in X$ converging to $x \in X$ implies $f(x_n) \to f(x)$ as $n \to \infty$. [5]

# B.Sc. (Honours) Examination-2025

Semester-VI

Mathematics

Course: CCMA - 14 [New]

(Algebra - IV)

**Time: 3 Hours** **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any six*** questions.

1. (a) Let $\phi : G_1 \to G_2$ be a group homomorphism. Let $H_1$ be a subgroup of $G_1$ and let $H_2$ be a subgroup of $G_2$. Prove that $\phi(H_1)$ is a subgroup of $G_2$ and $\phi^{-1}(H_2)$ is a subgroup of $G_1$.

   (b) State and prove that first isomorphism theorem of groups.

   (c) Prove that there exist exactly two groups of order 4. [3+4+3]

2. (a) Let $G$ be a finite abelian group of odd order. Prove that $\phi : G \to G$ defined by $\phi(x) = x^2$ is a group homomorphism. Justify whether $\phi$ is an isomorphism.

   (b) Let $\phi : \mathbb{R}^2 \to \mathbb{R}$ be defined by $\phi((x, y)) = x$. Prove that $\phi$ is a group homomorphism and determine $\ker(\phi)$.

   (c) Prove that $\mathbb{Z}/9\mathbb{Z}$ is not a homomorphic image of $\mathbb{Z}/16\mathbb{Z}$.

   (d) Let $G$ be an infinite group such that $G$ is a homomorphic image of $\mathbb{Z}$. Prove that $G \simeq \mathbb{Z}$. [3+3+2+2]

3. (a) Let $G$ be a group and let $\mathrm{Aut}(G) = \{\phi : G \to G : \phi \text{ is an isomorphism}\}$. Prove that $\mathrm{Aut}(G)$ is a group under the binary operation of composition of maps.

   (b) Let $G$ be a group such that $Z(G) = \{e\}$. Prove that $Z(\mathrm{Aut}(G)) = \{\mathrm{Id}_G\}$.

   (c) Prove that the characteristic subgroups of $(\mathbb{Q}, +)$ are precisely $\{0\}$ and $\mathbb{Q}$. [3+4+3]

4. (a) Let $R$ be a commutative ring with unity and let $I$ and $J$ be two ideals in $R$ such that $I + J = R$. Prove that $R/I \cap J \simeq R/I \times R/J$. Hence or otherwise find the smallest positive integer $n$ that leaves a remainder of 3 when divided by 7 and a remainder of 4 when divided by 11.

   (b) Prove that $\mathbb{Z}[i]/\langle 2 + i \rangle \simeq \mathbb{Z}/5\mathbb{Z}$. [6+4]

5. (a) Let $R$ be a commutative ring with unity and let $P$ be a non-zero prime ideal in $R$. Is $P[X] = \left\{ \sum_{i=0}^{m} a_i X^i : a_i \in P \text{ and } m \in \mathbb{N} \right\}$ necessarily a prime ideal in $R[X]$? Justify your answer.

   (b) Let $R$ be a commutative ring with unity and let $x \in R$ be a nilpotent element. Prove that $x$ belongs to every prime ideal of $R$.

(c) Let $R_1$ and $R_2$ be two commutative rings with unity and let $\phi : R_1 \to R_2$ be a ring homomorphism. Let $\mathfrak{q}$ be a prime ideal in $R_2$. Prove that $\phi^{-1}(\mathfrak{q})$ is a prime ideal in $R_1$.

(d) Let $R_1$ and $R_2$ be two commutative rings with unity and let $\phi : R_1 \to R_2$ be a ring homomorphism. Prove that $\phi(\text{Nil}(R_1)) \subseteq \text{Nil}(R_2)$. [4+2+2+2]

6. (a) Let $V$ and $W$ be finite-dimensional vector spaces over a field $F$ and let $T : V \to W$ be a linear transformation. Prove that $\dim(V) = \text{rank}(T) + \text{nullity}(T)$.

(b) Let $V$ be a vector space of dimension $n \geq 2$ over a field $F$ and let $T : V \to V$ be a linear transformation. If $T^2 \equiv 0$, then prove that $\text{rank}(T) \leq \frac{n}{2}$.

(c) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ and $U : \mathbb{R}^2 \to \mathbb{R}^3$ be linear transformations. Can $U \circ T : \mathbb{R}^3 \to \mathbb{R}^3$ be invertible? Justify your answer. [4+3+3]

7. (a) Let $S$ be a non-empty subset of a finite-dimensional vector space $V$. Prove that $S^0$ is a subspace of $V^*$. Moreover, if $S_1 \subseteq S_2 \subseteq V$, then prove that $S_2^0 \subseteq S_1^0 \subseteq V^*$.

(b) Let $W$ be a subspace of a finite-dimensional vector space $V$. Prove that

$$\dim(W) + \dim(W^0) = \dim(V).$$

(c) For two subspaces $W_1$ and $W_2$ of a finite-dimensional vector space $V$, prove that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0 \text{ and } (W_1 \cap W_2)^0 = W_1^0 + W_2^0.$$

[3+3+4]

8. (a) Prove that two similar matrices have the same eigenvalues.

(b) Let $V$ be a finite-dimensional vector space over a field $F$ and let $T : V \to V$ be a linear transformation. Let $\lambda \in F$ be an eigenvalue of $T$ and let $E_\lambda = \{\alpha \in V : T(\alpha) = \lambda\alpha\}$. Prove that $E_\lambda$ is a subspace of $V$.

(c) Let $\lambda \in F$ and $\mu \in F$ be two distinct eigenvalues of a linear transformation $T : V \to V$. Prove that $E_\lambda \cap E_\mu = \{0\}$.

(d) Prove that the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is not diagonalizable over $\mathbb{R}$ but is diagonalizable over $\mathbb{C}$. [2+2+2+4]

9. (a) Prove that $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ is not diagonalizable over $\mathbb{R}$ but $B = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is diagonalizable over $\mathbb{R}$.

(b) Let $A \in M_{n\times n}(\mathbb{R})$ be such that $A^k = 0$ for some integer $k \geq 1$. Prove that $A^n = 0$.

(c) Let $n > 2$ be an integer and let $A$ be an $n \times n$ real matrix such that $A^n = 0$ but $A^{n-1} \neq 0$. Prove that there does not exist any $n \times n$ real matrix $B$ such that $B^2 = A$. [5+3+2]

**B.Sc. (Honours) Examination-2025**
**Semester-VI (CBCS)**
**Mathematics**
**Course: DSEMA-3A**
**(Computer Fundamentals)**

**Time: Two Hours** **Full Marks: 20**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) Write an algorithm that reads five hundred (500) integers and write their sum of squares. [2]

   (b) Given the following declaration
   CHARACTER*30 SNAME*10, NAME*10, ADDRESS, CITY*5, MOBNUMBER*11
   how many character spaces are given to each of the declared variable? [1]

   (c) Considering the following declarations
   CHARACTER*6 NAME1='John', NAME2='Sammy'
   CHARACTER*7 NAME3='RITA', NAME4='MICHAEL'
   What is the result of following concatenation?
   (i) NAME2//NAME3
   (ii) NAME4//NAME3
   (iii) NAME1//NAME4
   (iv) NAME2//NAME4 [1]

   (d) Given a string A='12345678'. Find
   (i) A(:5) (ii) A(3+x:) , if x=2 (iii) A(2:8) [1]

2. (a) Draw a flow chart to reverse the digits of a three digit number. [2]

   (b) In the below program, find errors, if any, and then write the correct program.
   PROGRAM BUG
   REAL a,b,c
   a=b+c
   READ(*,*) c
   WRITE(*,*) a
   END PROGRAM SIMPLE [1]

   (c) Evaluate the following arithmetic expressions
   (i) 3**2 + 4/5
   (ii) 3 + 2.**3
   (iii) (3+2**3)/5
   (iv) 3**2**3 [1]

   (d) If A = 1.0, B = 2.5 and C = 6.5 find the value of the following logical expressions showing the order of evaluation.
   (i) A.GT.B.AND.A*B.LE.C
   (ii) A.GT.B.OR.A*B.LE.C
   (iii) NOT.A.GT.B.AND.A*B.LE.C
   (iv) A.GT.B.AND.(A.LE.C.OR.B.NE.C).OR.NOT.(A.EQ.B) [1]

3. (a) Draw a flowchart to find the summation
   $x - x^3/3! + x^5/5! - \ldots\ldots\ldots\ldots + (-1)^n x^{(2n+1)}/(2n+1)!$
   for given values of x and n. [2]

   (b) Assuming the declaration
   INTEGER MAT(3,3), NUM(6), I, J
   and the data line

1, 2, 3, 4, 5, 6, 7, 8, 9
find the value assigned to each array element in the following program segments:

(i)
```
DO 20 I=1,3
DO 20 J=1,3
IF (I.LT.J) THEN
MAT(I,J) = -1
ELSEIF(I.EQ.J) THEN
MAT(I,J) = 0
ELSE
MAT(I,J) = 1
ENDIF
20 CONTINUE
```

(ii)
```
DO 20 I=1,3
DO 30 J=1,3
30 MAT(I,J)=0
DO 10 J=I+1,J
MAT(I,J)=2
10 CONTINUE
20 CONTINUE
```

(iii)
```
READ(*,*) (NUM(k),k=1,6)
DO 20 I=1,3
DO 20 J=1,3
MAT(I,J) = NUM(I) + NUM(J)
20 CONTINUE
```

(iv)
```
READ(*,*) (NUM(N),N=1,6), (MAT(1,J),J=1,3)
DO 20 I=1,2
DO 20 J=1,3
MAT(NUM(I+1),NUM(J))=NUM(I+J)
20 CONTINUE
```

[1/2+1/2+1+1]

4. (a) The marks distribution of a course is: Attendance - 10%, Class Tests - 20%, Midterm Exam - 20% and Final Exam - 50%. The grades given are: $A$ for $\geq 90\%$, $B$ for $\geq 80\%$ and $< 90\%$, $C$ for $\geq 70\%$ and $< 80\%$, $D$ for $\geq 60\%$ and $< 70\%$ , $F$ for $< 60\%$. If the teacher takes 40 classes and if Class Tests, Midterm and Final Exam have full marks of 100 each, write a FORTRAN program to calculate a student's grade. [3]

   (b) Write down the FORTRAN expression of the following mathematical expressions:

   (i) $cos\left(\frac{(a^5+5|d|+log_{10}|d|)}{cot(b^{-2}-c^{-3})cosec^{-1}(x)}\right)$ (ii) $cot^{-1}\left(\frac{1}{\sqrt{2k}}\right)e^{-\frac{(x-c)^2}{2k^2}}$ (iii) $\frac{ab^c+bc^a}{c+\frac{d}{|a+ib|}}$ [1+1/2+1/2]

5. Describe the execution of a DO statement. Elaborate the execution using flow chart. [3+2]

6. (a) The horizontal ($x$) and vertical ($y$) distances traveled by a cricket ball are given by

$$x = utcos\alpha \quad (1)$$

$$y = utsin\alpha - \frac{gt^2}{2}. \quad (2)$$

   Write a FORTRAN program to do the following,
   (i) Read $u, \alpha, g$ and $x$ .
   (ii) Calculate $t$ from equation (1), $y$ from equation (2) and write $t$ and $y$ to the screen.
   (iii) Write 'Big Six' if $y \geq 8$, 'Maybe Six' if $0 \leq y \leq 8$, 'Not Six' otherwise. [3]

   (b) Write two differences between STOP and END statements. [1]

   (c) Define a statement function to calculate $R = \sqrt{u^2+v^2+w^2}$ and write assignment statements to calculate $A = \frac{x}{\sqrt{x^2+y^2+z^2}}$, $B = \sqrt{x^4+y^4+z^4}$, $C = \sqrt{4x^2+9y^2+4z^2}$ using that statement function. [1]

Use separate answer scripts for each group

# B. Sc. (Honours) Examination-2025

**Semester-VI (CBCS)**
**Mathematics**
**Paper: DSEMA-4**
**(Mathematical Modelling)**

**Time: 3 Hours** **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

## Group A (Physical Systems)
**(Full Marks: 30)**
Answer ***any five*** questions.

1. (a) Define Radioactive decay. A sample contains 1 gram of a radioactive isotope with a half-life of 10 days. How much of the isotope remains after 25 days and what is its activity if the decay constant is $\lambda = 0.0693\,\text{days}^{-1}$?
   (b) The half-life of a radioactive isotope is 12 hours. If you start with a sample of 100 grams, how much remains after 36 hours? [(1+3)+2]

2. Obtain the simple harmonic motion (SHM) for modelling a spring mass system. Convert the equation into a system of ordinary differential equations of the form $\dot{X} = AX$. Find the eigenvalues of the Matrix $A$ and state the nature of the fixed point. [1+2+2+1]

3. What is continuously compound interest? After a working life of careful saving and investing, your pension fund now totals \$400000. Your plan is to take out an investment that pays 2.0% interest annually, continuously compounded and to withdraw \$25000 per year at a constant rate through the year. Assuming the interest rate does not change, how long do you expect the money to last? [1+5]

4. (a) What restrictions are used to model the heat equation?
   (b) What is a wave equation? Explain the different boundary conditions may arise with the equation. [3+3]

5. (a) Define Kirchhoff's voltage law, Ohm's law and Faraday's law.
   (b) A series LRC circuit has $L = 0.1\,H$, $R = 20\,\Omega$, $C = 0.0001\,F$, initial charge $q(0) = 0.01\,C$, initial current $\frac{dq}{dt}(0) = 0\,A$ with no external voltage ($V(t) = 0$). Find $q(t)$ and relate it to SHM. [3+3]

6. Write a short note on the following:
   (a) Discrete and continuous models
   (b) Deductive, inductive and floating models
   (c) Explicit and implicit models. [2+2+2]

7. A 1000 L tank of water initially contain 10 kg of dissolved salt. A pipe brings a salt solution (concentration 0.005 kg/L) into the tank at a rate 2 L/s and a second pipe carries away the excess solution at the same rate.
   (a) Calculate $c(t)$, the concentration of the salt in the tank, assuming the tank is well mixed.
   (b) After how many minutes has the concentration of salt decreased 25% of its initial value?
   (c) What happens as '$t$' tends to infinity? [2+2+2]

8. Convert the following system into polar coordinates and show that the system undergoes a limit cycle:

$$\frac{dx}{dt} = y + \frac{x}{\sqrt{x^2+y^2}}\{1-(x^2+y^2)\},$$
$$\frac{dy}{dt} = -x + \frac{y}{\sqrt{x^2+y^2}}\{1-(x^2+y^2)\}.$$

[3+3]

## Group B (Biological Systems)
**(Full Marks: 30)**

Answer question number 1 and ***any two*** from the rest.

1. Answer ***any two*** questions.

(a) How do you interpret stability-instability dynamics for a continuous one-dimensional autonomous system at an equilibrium point?

Suppose a single infected individual migrates into a community containing $N$ individuals susceptible to a disease. The infected individual spreads the disease to all susceptible. Let $S(t)$ be the number of susceptibles at time $t$ and satisfy the following equations:

$$\frac{dS}{dt} = -rS(N+1-S),$$
$$S(t=0) = N,$$

where $r$ is a positive constant which measures the infection. Obtain the solution of the model and comment biologically for a long span of time.

(b) Explain phase plane analysis for the following autonomous system:

$$\frac{du}{dt} = f(u,v),$$
$$\frac{dv}{dt} = g(u,v).$$

Sketch the phase diagram and classify the equilibrium point for the linear autonomous system

$$\frac{du}{dt} = -u+3v,$$
$$\frac{dv}{dt} = -6u+5v.$$

(c) Explain symbiosis and mutualism mechanism in ecological environment. Develop a two-species Lotka-Volterra model of mutualism mechanism and hence show that the system cannot have limit cycles. [2 × 5=10]

2. (a) (i) How do you explain the fixed point of a map $f : \mathbb{R} \to \mathbb{R}$ in the context of flow in a discrete system?
(ii) Explain linear stability analysis around the fixed point $x^*$ of the map $x_{n+1} = f(x_n)$ where $f : \mathbb{R} \to \mathbb{R}$ and $x_n$ denotes the $n$-th iteration.
(iii) Consider the map $f(x) = 1-\mu x^2$, where $x \in [-1,1]$ and $\mu \in [0,2]$. Locate every fixed point on the map. Find out their stability characters as well.

(b) A population follows a generalized logistic growth given by

$$\frac{du}{dt} = \frac{ru}{\alpha}\left[1-\left(\frac{u}{K}\right)^{\alpha}\right],$$
$$u(t=0) = u_0 \geq 0.$$

(i) Find the exact solution of the differential equation for $r, \alpha > 0$ and show that the limiting population is still $K$.
(ii) What happens to the model if $\alpha \to 0$ and $\alpha \to -1$?

3. Construct [(1+2+3)+(3+1)]

$$\mathcal{L} = \left[u-u^*-u^* log_e\left(\frac{u}{u^*}\right)\right] + \frac{\gamma}{\beta}\left[v-v^*-v^* log_e\left(\frac{v}{v^*}\right)\right]$$

about the coexistence equilibrium point $(\frac{\delta}{\gamma}, \frac{\alpha}{\beta})$ for the Lotka-Volterra predator-prey model

$$\frac{du}{dt} = \alpha u - \beta uv,$$
$$\frac{dv}{dt} = \gamma uv - \delta v,$$
$$u(t=0) \geq 0,\ v(t=0) \geq 0,$$

where $\alpha$, $\beta$, $\gamma$ and $\delta$ are all positive. Is the system conservative? Justify your answer. [5+3+2]

4. Explain a reaction-diffusion system with an example.
Consider the following reaction-diffusion system:

$$\frac{\partial u(x,t)}{\partial t} = R(u) + D\frac{\partial^2 u(x,t)}{\partial x^2}.$$

(a) Explain the reaction-diffusion model mathematically.
(b) Calculate the values of $D$ for which the diffusion-driven instability about the non-trivial equilibrium point can take place.

[2+3+5]