B. A./B. Sc. Examination-2020

Semester-II(CBCS)

Mathematics

Generic Elective: GEC-2

(Introductory Algebra(Theory))

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any six*** questions.

1. (a) Let z_1, z_2 be two non-zero complex numbers. If θ_1 be an argument of z_1 and θ_2 be an argument of z_2 then prove that $\theta_1 - \theta_2$ is an argument z_1/z_2.
   (b) Prove that $\sqrt{i} + \sqrt{-i} = \sqrt{2}$.
   (c) Find the sum of 99th powers of the roots of the equation $x^7 - 1 = 0$.
   (d) Expand $\cos^7\theta$ in a series of cosines of multiples of θ.

2. (a) Prove that $x^2 + x + 1$ is a factor of $x^{10} + x^5 + 1$.
   (b) Find the remainder when $x^4 + 2x^3 + 1$ is divided by $(x-1)(x-2)$.
   (c) Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$, if it's roots are in geometric progression(GP).
   (d) Find the equation whose roots are the roots of the equation $x^3 + 5x^2 - 7x + 2 = 0$ each increased by 1.

3. (a) Solve the equation $x^4 - 4x^3 - 4x^2 - 4x - 5 = 0$ given that two roots α, β are connected by the relation $2\alpha + \beta = 3$.
   (b) If α, β, γ,δ be the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find the values of

   (i) $\sum \alpha\beta/\gamma$,
   (ii) $\sum \alpha^2/\beta$.

[2-

4. (a) Express the following matrix as the sum of a symmetric matrix and a skew-symmetric matrix,

$$\begin{pmatrix} 0 & 5 & 2 & 3 \\ 0 & 7 & 2 & 2 \\ 8 & 7 & 2 & 6 \\ 1 & 4 & 5 & 0 \end{pmatrix}.$$

   (b) Define row-reduced echelon matrix. Apply elementary row operations to reduce the following matrix to a row reduced echelon matrix

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}.$$

[1+

(c) Suppose A be an $n \times n$ real skew-symmetric matrix and P be an $n \times 1$ real matrix. Prove that $P^tAP = 0$.

5. (a) Define rank of a matrix. Find all real value(s) of λ for which rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & \lambda \\ 5 & 7 & 1 & \lambda^2 \end{pmatrix}$ is (i)2 and (ii)3. [1+2

(b) Consider the following system of linear equations,

$$x - 2y + 3z = 1,$$
$$2x - 5y + 10z = 0,$$
$$4x - 7y + 8z = 6.$$

Write down the matrix representation of the above system of linear equations. Solve the above system of linear equations by Gaussian elimination method. [3]

(c) Write down the vector $v = (1, -2, 5)$ as a linear combination of the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$ and $u_3 = (2, -1, 1)$ [4]

6. (a) Define eigenvalue and eigenvector of a square matrix. Find eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$. [1+4]

(b) Let A be a non-singular matrix and λ be a non-zero eigen value of A. Then show that λ^{-1} is a eigen value of A^{-1}. [3]

(c) Prove that a square matrix A is singular if and only if 0 is an eigen value of A. [2]

7. (a) Let $v_1 = \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix}, v_2 = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix}, w = \begin{pmatrix} 1 \\ 14 \\ -9 \end{pmatrix}$. Determine if w is in $\text{col}(A)$, where $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$. [4]

(b) Determine whether $\{u_1, u_2, u_3\}$ forms a basis of R^3. where $u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. [4]

(c) Show that the set H of all points in R^2 of the form $(3s; 2 + 5s)$ is not a vector space, by showing that it is not closed under scalar multiplication. (Find a specific vector u in H and a scalar c such that cu is not in H.) [2]

8. (a) Verify the parallelogram law for vectors u and v in R^n: $\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2$.

[3]

(b) Consider vectors $u = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, v = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, w = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$ in R^3. Find

1. $u \cdot v$ 1
2. $v \cdot w$ 1
3. $\|u\|$ 1

[1+1+1]

(c) Show that v_1, v_2, v_3 is an orthonormal basis of R^3, where $v_1 = \begin{pmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{pmatrix}, v_2 = \begin{pmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{pmatrix}, w = \begin{pmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{pmatrix}$

9. (a) Verify the set $S = \{u_1, u_2, u_3\}$ is a orthogonal basis of R^3. Then express x as a linear combination of the vectors in S. Where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -1/2 \\ -2 \\ 7/2 \end{pmatrix}, x = \begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$. [2+

(b) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of R^4 spanned by $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$. [4+

Use separate answer scripts for each unit

B.Sc.(Honours) Examination, 2020

Semester-II (CBCS)
(For Back Candidates)
Mathematics (Honours)
Core Course : CC-3
(Analysis-II and Algebra-II)

Time: Three Hours **Full Marks: 60**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I [Analysis-II (Marks-30)]

Answer ***Question no.1*** and ***any four*** from the rest

[2 × 5]

1. Answer any five questions
   (a) Find closure of the sets $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$.
   (b) Find the limit points of the sets $(0,1) \cup (1,2) \cup (2,3) \cdots$.
   (c) Define lim sup and lim inf of the sequence $\{x_n\}$.
   (d) Give example with justification of two sets which are both open and closed.
   (e) Show that arbitrary intersection of open sets may not be an open set.
   (f) Find asymptote of the curve $x^3 + y^3 - 3axy = 0$.
   (g) Find area and total length of a circle of radius a.

2. State and prove Taylor's theorem with Cauchy's form of remainder. [5]

3. Find total length of perimeter of the cardioid $r = a(1 + cos\theta)$. [5]

4. Expand $(1+x)^{-1}$ in power series with proper justtification [5]

5. State and prove Heine Borel theorem. [5]

6. Show that closure of the set A, $\overline{A} = A \cup \acute{A}$ where $\acute{A}$ is derived set of A. [5]

7. A circle moves with its centre on the parabola $y^2 = 4ax$ and always passes through the vertex of the parabola. Show that the envelope of the circle is the curve $x^3 + y^2(x + 2a) = 0$ [5]

Unit - II [Algebra-II(Marks-30)]

Answer ***any three*** questions.

8. (a) If a is an odd integer, show that $24|a^2(a^2-1)$. [3]
   (b) Find integers u and v satisfying $52u - 91v = 78$. [4]
   (c) Find the primes of the form $n^2 - 4$, n being an integer. [3]

9. (a) Find the remainder when $1! + 2! + \cdots + 50!$ is divided by 15. [3]
   (b) Find the least positive integer that leaves remainders 2, 3 and 2 when divided by 3, 4 and 5 respectively. [5]
   (c) Find the units digit in 7^{99}. [2]

10. (a) Show that the set $\{5, 15, 25, 35\}$ is a group under multiplication modulo 40. Can you see any relationship between this group and U_8? [4+2]
    (b) For any elements a and b from a group and any integer n, prove that $(a^{-1}ba)^n = a^{-1}b^n a$. [4]

11. (a) In a finite group G of even order show that there exists an odd number of elements of order 2. [3]
    (b) Give an example of a group $(G, \cdot)$ in which $O(a)O(b) \neq O(a \cdot b)$ [2]
    (c) Let H and K be two subgroups of a group G. Then prove that HK is a subgroup of G if and only if $HK = KH$. [5]

12. (a) If an Abelian group G of order 10 contains an element of order 5, prove that G must be cyclic. [3]
    (b) Let G be a cyclic group of order 30 generated by a. Find a subgroup H of order 6 and generators of H. [3+2]
    (c) Determine all finite subgroups of R^*. [2]

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(For Back Candidates)
Mathematics (Honours)
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(Analysis-II and Algebra-II)

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[2 × 5]

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Unit - II [Algebra-II(Marks-30)]

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B. Sc. (Honours) Examination-2020

Semester-II(CBCS)
Mathematics (Honours)
Core Course : CC-4
(Geometry and Vector Calculus)

Time: 3 Hours **Full Marks: 60**

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Unit-I [Geomerty (Marks: 30)]

Answer **Question No. 1** and ***any two*** from the rest.

1. Answer any five questions: $5X2 = 10$

   (a) Find the transformed equation of the straight line $\frac{x}{a} + \frac{y}{b} = 2$ when the origin is transfered to the point (a, b).

   (b) Find the eccentricity and foci of the ellipse $5x^2 + 9y^2 - 40x - 36y = 9$.

   (c) Find the points of intersection of the two conics $\frac{1}{r} = 1 + cos\theta$ and $\frac{3}{r} = 1 - cos\theta$.

   (d) Find the radius of the circle $3x^2 + 3y^2 + 3z^2 + x - 5y - 2 = 0, \quad x + y = 2$.

   (e) Find the equation of the tangent plane to the hyperbolic paraboloid $5x^2 - 2y^2 = 2z$ parallel to $10x - 6y - z = 7$.

   (f) Show that the angle between the lines of section of plane $3x + y + 5z = 0$ and the cone $6yz - 2zx + 5xy = 0$ is $\cos^{-1}\frac{1}{6}$.

   (g) On the conic $r = \frac{6}{1-cos\theta}$, find the point with least radius vector.

2. (a) When the axes are turned through an angle, the expression $ax + by$ becomes $a'x' + b'y'$ referred to new axes; show that $a^2 + b^2 = a'^2 + b'^2$. [5]

   (b) Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its canonical form and determine the nature of the conic. [5]

3. (a) The normals at the ends of the locus rectum of the parabola $y^2 = 4ax$ meet the carve again in Q and Q'. Prove that $QQ' = 12a$. [5]

   (b) Prove that the straight line $rcos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + ecos\theta$ if $(l\ cos\alpha - ep)^2 + l^2 sin^2\alpha = p^2$. [5]

4. (a) Find the equation of the sphere passing through the point $(1, 0, -3)$ and through the circle represented by $x^2 + y^2 + z^2 - 4x - 6y + 2z = 16, \quad 3x + y + 3z - 4 = 0$. [4]

   (b) If the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, the find the equation of the other two. [6]

5. (a) If P, Q, R and P', Q', R' be the feet of the six normals drawn from a point to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and the plane PQR is given by $lx + my + nz = p$ then show that the equation of the plane $P'Q'R'$ is given by $\frac{x}{a^2 l} + \frac{y}{b^2 m} + \frac{z}{c^2 n} + \frac{1}{p} = 0$. [6]

   (b) If the normal at a point P of then hyperboloid $x^2 + y^2 = 4z$ meets the plane YOZ and ZOX at the points Q and R respectively, then show that $PQ = PR$. [4]

Unit-II [Vector Calculus (Marks - 30)]

Answer ***any five*** questions.

1. (a) Show that if $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be any three non-coplanar vectors then $\mathbf{b} \times \mathbf{c}$, $\mathbf{c} \times \mathbf{a}$, $\mathbf{a} \times \mathbf{b}$ are also non-coplanar. Express $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in terms of $\mathbf{b} \times \mathbf{c}$, $\mathbf{c} \times \mathbf{a}$ and $\mathbf{a} \times \mathbf{b}$. [1+3]

   (b) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ by vector method. [2]

2. (a) If $\mathbf{a}$ and $\mathbf{b}$ are two non-collinear vectors such that $\mathbf{a} = \mathbf{c} + \mathbf{d}$, where $\mathbf{c}$ is a vector parallel to $\mathbf{b}$ and $\mathbf{d}$ is a vector perpendicular to $\mathbf{b}$, obtain expressions for $\mathbf{c}$ and $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

   (b) Find $\mathbf{r}$ and λ, where $\mathbf{a} \times \mathbf{r} = \mathbf{b} \times \lambda\mathbf{a}$; $\mathbf{a}.\mathbf{r} = 2$. Given that $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. [3]

3. (a) In the triangle $\triangle ABC$ let $\mathbf{BC} = \mathbf{a}$, $\mathbf{CA} = \mathbf{b}$, $\mathbf{AB} = \mathbf{c}$. Prove that $\cos C = \frac{a^2+b^2-c^2}{2ab}$ by vector method, where a, b, c denote the lengths of sides of the triangle. [4]

   (b) Find the equation of a plane containing the line $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and parallel to the vector $\mathbf{c}$. [2]

4. (a) If α and β are perpendicular to each other, then prove that $\alpha \times (\beta \times \gamma)$ and $(\alpha \times \beta) \times \gamma$ are also perpendicular to each other. [3]

   (b) Show that a necessary and sufficient condition that a proper vector $\mathbf{u}$ always remains parallel to a fixed line is that $\mathbf{u} \times \frac{\mathrm{du}}{\mathrm{dt}} = \mathbf{0}$. [3]

5. (a) Find the curvature of the curve $\mathbf{r} = \mathbf{f}(t)$ by vector method. [3]

   (b) Prove that $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$. [3]

6. (a) Prove that $\int_\Gamma \mathbf{t}.d\mathbf{r} = 2\pi$, where $\mathbf{t}$ is the unit tangent vector and Γ is the unit circle in the xy-plane about the origin as center. [3]

   (b) Determine whether the force field $\mathbf{F} = 2xz\mathbf{i} + (x^2 - y)\mathbf{j} + (2z - x^2)\mathbf{k}$ is conservative or not. [3]

7. Verify Green's theorem in the plane for $\oint_\Gamma (x^2dx + xydy)$, where Γ is the square in the xy-plane given by $x = 0$, $y = 0$, $x = a$, $y = a$ $(a > 0)$ described in the positive sence (anti-clockwise manner). [6]

8. Verify the Divergence Theorem for the vector function $\mathbf{F} = (x^2 - yz)\mathbf{i} + (y^2 - zx)\mathbf{j} + (z^2 - xy)\mathbf{k}$, taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. [6]